

MAT 534 — HOMEWORK 12

DUE ON FRIDAY 25 APRIL

1. (Ch. 14, #6) Find all maximal ideals in \mathbb{Z}_{10} and \mathbb{Z}_{12} . (Remember that an ideal is in particular an additive subgroup, and we know all the subgroups of \mathbb{Z}_n .)
2. (Ch. 14, #9) Let $n \geq 1$. Prove that the ideal $n\mathbb{Z}$ of \mathbb{Z} is a prime ideal if and only if n is a prime number.
3. (Ch. 14, #17) Let R be a commutative ring and I an ideal of R . If I contains a unit u of R , prove that $I = R$.
4. (Ch. 14, #26) Let R be a commutative ring with identity and I an ideal of R . Prove that R/I is also commutative and has an identity.
5. (Ch. 14, #44) Let R be a commutative ring, I an ideal of R , and $a \in R$. Prove that the set $(I : a) = \{x \in R \mid ax \in I\}$ is an ideal of R .