MAT 534 — HOMEWORK 10

DUE ON MONDAY 7 APRIL

1. (Ch. 10, #19) Let $\varphi\colon \mathbb{Z}_{17} \longrightarrow G$ be a homomorphism which is not one-to-one. Prove that the kernel of φ is all of $\mathbb{Z}_{17}$. What magical property of the number 17 did you use?

2. (Not in text, but exactly like something we did in class.) Let $\varphi\colon \mathbb{Z}_n \longrightarrow G$ be a homomorphism. Prove that φ is completely determined by $\varphi(1)$.

3. (Ch. 10, #26) Determine all homomorphisms $\varphi\colon \mathbb{Z}_4 \longrightarrow \mathbb{Z}_2 \oplus \mathbb{Z}_2$. (Hint: previous problem.)

4. (Ch. 10, #27) Determine all homomorphisms $\varphi\colon \mathbb{Z}_n \longrightarrow \mathbb{Z}_n$.

5. (Ch. 10, #56) Define $\varphi\colon \mathbb{R} \longrightarrow \mathrm{GL}_2(\mathbb{R})$ by

$$\varphi(t) = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}.$$

Prove that φ is a homomorphism, and find its kernel.

6. (Ch. 11, #5) Prove that any Abelian group of order 45 has an element of order 15. Must an Abelian group of order 45 have an element of order 9?

7. (Ch. 11, #10) Find all Abelian groups (up to isomorphism) of order 360.

8. (Ch. 11, #15) How many Abelian groups (up to isomorphism) are there

(a) of order 6?

(b) of order 42?

(c) of order pq, where p and q are distinct primes?

Justify.