MAT 534 — HOMEWORK 9

DUE ON MONDAY 31 MARCH

1. (Ch. 9, #34) In $\mathbb{Z}$, let $H = \langle 5 \rangle$ and $K = \langle 7 \rangle$. Prove that $\mathbb{Z} = H + K$. (This is the additive version of the condition "$G = HK$".) Is $\mathbb{Z}$ the internal direct product of H and K?

2. (Ch. 9, #51, 55) Let G be a group and $H < K < G$ subgroups. Prove the following statements.

(a) If H is normal in G, then H is normal in K.

(b) If H is normal in G, then K/H (the set of cosets of H in K) is a subgroup of G/H.

(c) If H is normal in G and K is normal in G, then K/H is a *normal* subgroup of G/H.

(d) Normality is *not* transitive: it's possible to have H normal in K, and K normal in G, but H not normal in G. (Consider $G = D_4$, $K = \{R_0, R_{180}, D, D'\}$, and $H = \{R_0, D\}$.)

3. (Ch. 9, #65) If G is a non-Abelian group, then Aut(G) is not cyclic. (Combine three theorems.)

4. (Ch. 10, #9 and #13) Let G and G' be groups, and prove that $(G \oplus G')/(\{e\} \oplus G') \cong G$. (Define a function $G \oplus G' \longrightarrow G$, show that it's a homomorphism, and determine its kernel. Then use the First Isomorphism Theorem.)

5. (Ch. 10, #14) Explain why the rule $\varphi(k) = 3k$ does *not* define a homomorphism from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{10}$.