MAT 534 — HOMEWORK 4

DUE ON FRIDAY 7 FEBRUARY

All these problems are from Chapter 5.

1. (#1) Let

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \tau = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}.$$

Compute σ^{-1}, $\tau\sigma$, and $\sigma\tau$.

2. (#3) Write each of the following permutations as a product of disjoint cycles.

(a) (1235)(413)

(b) (13256)(23)(46512)

(c) (12)(13)(23)(142)

3. (#4) Find the orders of the following permutations:

(a) (14)

(b) (14762)

(c) (124)(357)

(d) (124)(357869)

4. (#10) What is the maximum order of an element of A_{10}?

5. (#17) Prove Theorem 5.6, that the set A_n of even permutations in S_n forms a subgroup.

6. (#29) How many elements of order 4 does S_6 have? How many elements of order 2 does S_6 have?

7. (#31) Let G be a group of permutations of a set X. Let $a \in X$ and define stab(a) to be the set of permutations $\sigma \in G$ such that $\sigma(a) = a$. This is called the *stabilizer* of a. Prove that it is a subgroup of G.

Also suggested, but not to hand in:

(A) (#27) Use Table 5.1 on page 111 (the Cayley table for the group A_4) to compute the following:

(a) the centralizer of $\alpha_3 = (13)(24)$.

(b) the centralizer of $\alpha_{12} = (124)$.

(B) (#42) In S_3, find elements α and β such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha\beta| = 3$.

(C) (#45) Prove that S_n is non-Abelian for all $n \geq 3$.

(D) (#72) Let G be a group. Prove or disprove: The set $H = \{g^2 \mid g \in G\}$ is a subgroup of G.