MAT 534 — HOMEWORK 3

DUE ON FRIDAY 31 JANUARY

All these problems are from Chapter 4.

1. (#1 and 2) Find all generators of $\mathbb{Z}_8$. If $G = \langle a \rangle$ is a cyclic group of order 8, find all generators of G.

2. (#9) How many sugroups does $\mathbb{Z}_{20}$ have? List a generator for each.

3. (#10) Let $G = \langle a \rangle$ be a cyclic group of order 24. We know G has a unique subgroup H of order 8, and H is cyclic. Find all generators for H.

4. (#21(c)) Let G be a cyclic group of order 24, and let a be an element of G. If $a^8 \neq e$ and $a^{12} \neq e$, prove that $\langle a \rangle = G$.

5. (#22) Prove that a group of order 3 is cyclic.

6. (#39) Give an example of a group that has exactly 6 subgroups (including the trivial and improper subgroups).

7. (#40) Let m and n be integers. We know that the intersection of the subgroups $\langle m \rangle$ and $\langle n \rangle$ is a subgroup of $\mathbb{Z}$, so is cyclic. Find a generator.

Also suggested, but not to hand in:

(A) (#14) Suppose that a cyclic group G has exactly three subgroups: G itself, $\{e\}$, and a subgroup of order 7. What is $|G|$? What can you say if 7 is replaced with p, an arbitrary prime integer?

(B) (#30) Suppose a and b are elements of a group, a has odd order, and $aba^{-1} = b^{-1}$. Show that $b^2 = e$.

(C) (#36) Prove that a finite group is the union of proper subgroups if and only if the group is not cyclic.

(D) (#37) Prove that the group of positive rational numbers under multiplication is not cyclic.