MAT 534 — HOMEWORK 2

DUE ON FRIDAY 24 JANUARY

1. (Chapter 2, #32) Construct a Cayley table for $U(12)$.

2. (Chapter 3, #4) Prove that in any group, an element a and its inverse a^{-1} have the same order. (Make sure you prove they're *equal*, not just that $|a^{-1}| \le |a|$.)

3. (Chapter 3, #10) How many subgroups of order 4 does D_4 have?

4. (Chapter 3, #18) Suppose $a^6 = e$. What are the possible orders of a?

5. (Chapter 3, #22) Show that $U(14) = \langle 3 \rangle$. (Hence, $U(14)$ is cyclic.) Is $U(14) = \langle 11 \rangle$?

6. (Chapter 3, #34) Prove that $C(a) = C(a^{-1})$ for every element a of a group. (Here $C(-)$ denotes the centralizer. Make sure to prove both containments.)

7. (Chapter 3, #37) Suppose G is the group defined by the following Cayley table.

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	8	7	6	5	4	3
3	3	4	5	6	7	8	1	2
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	4	3	2	1	8	7
7	7	8	1	2	3	4	5	6
8	8	7	6	5	4	3	2	1

(a) Find the centralizer of the elements 1, 2, 6 of G.

(b) Find $Z(G)$.

(c) Find the order of each element of G.

Also suggested, but not to hand in:

(A) (Chapter 3, #1) For each of the following groups, find the order of the group and the order of each element: $\mathbb{Z}_{12}$, $U(12)$, D_4.

(B) (Chapter 3, #49) Suppose a group contains elements a and b such that $|a| = 4$, $|b| = 2$, and $a^3 b = ba$. Find $|ab|$.