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I will grade problems 1, 2, and 7 myself. (4) (6) (4)

③ (i) The identity $G \to G$ is an isomorphism. ✓

(ii) If $\varphi : G \to H$ is an isom, then $\varphi^{-1} : H \to G$

6 is also one-one and onto. Also, if $\varphi(g) = x$ and $\varphi(h) = y$, then $\varphi(gh) = xy$, so

$$\varphi^{-1}(xy) = \varphi^{-1}(x)\,\varphi^{-1}(y)$$

and φ^{-1} is an isomorphism. ✓

(iii) If $\varphi : G \to H$ and $\psi : H \to K$ are isoms, then $\psi \circ \varphi : G \to K$ is one-one and onto, and for $g, h \in G$

$$\begin{aligned}(\psi \circ \varphi)(gh) &= \psi(\varphi(gh)) \\ &= \psi(\varphi(g)\,\varphi(h)) \\ &= \psi(\varphi(g))\,\psi(\varphi(h)).\ (\psi\circ\varphi\end{aligned}$$

so $\psi \circ \varphi$ is an isom. ✓

④ ($\Rightarrow$) If φ is an automorphism then for all g, h $\varphi(gh) = \varphi(g)\varphi(h)$, that is, $(gh)^{-1} = g^{-1}h^{-1}$.

4 But then $gh = hg$ for all g, h, so G is Abelian. ✓

($\Leftarrow$) If $g^{-1}h^{-1} = h^{-1}g^{-1}$ for all h, g then φ respects the operation. Easy to check it's 1-1 and onto. ✓

⑤ Define $\varphi\left(\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\right) = a \in \mathbb{R}$. Check it's an isom.

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6 a) showed already in #3

b) Let $H \leq G$ and $\varphi: G \longrightarrow G'$ an

4 isom. Use a Subgroup Test:

$\varphi(H)$ is nonempty since $e' = \varphi(e) \in \varphi(H)$.

If $\varphi(h), \varphi(h') \in \varphi(H)$ then

$$\varphi(h)\varphi(h') = \varphi(hh') \in \varphi(H)$$

and $\varphi(h)^{-1} = \varphi(h^{-1}) \in \varphi(H)$ ✓.