

HW 4 Solutions.

① $\sigma^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$

⑥ $\tau\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$
 $\sigma\tau = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}.$

(no justification needed)

② (a) $(15)(234)$

⑥ (b) $(124)(35)(6)$ (ok to omit (6)).

(c) (1423)

(again no justification needed)

③ (a) $|(14)| = 2$

(b) $|(14762)| = 5$

⑧ (c) order 3

(d) order 6

④ Check all cycle types - the largest lcm of the lengths of cycles in A_{10} is $7 \cdot 3 = 21$

⑤ Use one of the Subgroup Tests (1-step, 2-step, or Finite).

⑥ Two cycle types of order 4: $4, 1, 1$, and $4, 2$. There are $\binom{6}{4} = 15$ sets of indices in the 4-cycle, and $3! = 6$ different 4-cycles

on the indices $(1, 2, 3, 4)$. So $15 \cdot 6 = 90$
4-cycles. So in all there are
180 elements of order 4.

Cycle types of order 2:

$2, 2, 2$

$2, 2$

2

number

$$\binom{6}{2} \binom{4}{2} = 90$$

$$\binom{6}{2} \binom{4}{2} = 90$$

$$\binom{6}{2} = 15$$

$$\hline 195$$

(this one is hard).

④ ⑦ Use a Subgroup Test.

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