534 HW #3

(1) generators of $\mathbb{Z}_8 = \{1, 3, 5, 7\}$ by Cor 4
4 generators of $\langle a \rangle = \{a, a^3, a^5, a^7\}$ by Cor 3

(2) $\mathbb{Z}_{20}$ has subgroups $\langle 1 \rangle, \langle 2 \rangle, \langle 4 \rangle, \langle 5 \rangle,$
3 $\langle 10 \rangle, \langle 20 \rangle$ by Thm 4.3

(3) $H = \langle a^3 \rangle$ by Thm 4.3. Generators
6 are $\{a^3, (a^3)^3, (a^3)^5, (a^3)^7\}$
13 by Cor 3.

(4) Since $a^8 \neq e \neq a^{12}$, $|a|$ does not divide
4 8 or 12. But it does divide 24.
So $|a| = 24$.

(5) S'pose $G = \{e, a, b\}$. If $a^2 = b$, we're
5 done, so we must have $a^2 = e$.
But then $|a|$ does not divide $|G|$, ⨳.

(6) See #2 above
3

(7) Claim $\langle m \rangle \cap \langle n \rangle = \langle \mathrm{lcm}(m,n) \rangle$.
We know "$\supseteq$" since $\mathrm{lcm}(m,n)$ is a
6 multiple of both m and n. If $k \in \mathbb{Z}$
is a mult of m and n then k is
31 divisible by $\mathrm{lcm}(m,n)$, done.