MAT 631 — HOMEWORK 2

DUE ON TUESDAY 10 SEPTEMBER

1. Suppose that $a^2 = 1$ for every element a in a group G. Prove that G is Abelian.

2. Consider the presentation

$$D_{2n} = \langle \sigma,\ \tau \mid \sigma^n = 1,\ \tau^2 = 1,\ \sigma\tau = \tau\sigma^{-1} \rangle .$$

(a) Show that D_{2n} is generated by two element of order 2, namely τ and $\tau\sigma$.

(b) Deduce that D_{2n} has another presentation

$$D_{2n} = \langle a,\ b \mid a^2 = b^2 = (ab)^n = 1 \rangle .$$

(Derive the relations on $a = \tau$ and $b = \tau\sigma$ from those on σ and τ, and vice versa.)

3. Define permutations

$$\sigma: \qquad 1 \mapsto 3,\ 2 \mapsto 4,\ 3 \mapsto 5,\ 4 \mapsto 2, 5 \mapsto 1$$

$$\tau: \qquad 1 \mapsto 5,\ 2 \mapsto 3,\ 3 \mapsto 2,\ 4 \mapsto 4,\ 5 \mapsto 1$$

Find the cycle decompositions of σ, τ, σ^2, τ^2, $\sigma\tau$, and $\tau\sigma$.

4. Find the order of the permutation (1 8 10 4)(5 7)(6 9).

5. Prove that the order of a permutation σ is the least common multiple of the lengths of the cycles in its cycle decomposition. Find all n such that S_5 contains an element of order n.

6. (bonus problem) Consider

$$Y = \langle u,\ v \mid u^4 = v^3 = 1,\ uv = v^2u^2 \rangle .$$

Prove that Y is the trivial group. (Hint: consider uvu^2 and vuv.)