MAT 631 — HOMEWORK 12

DUE ON TUESDAY 4 DECEMBER

1. Let $\langle \mid \rangle$ be a symmetric bilinear form on a vector space V over a field F. Define the *quadratic form* associated to $\langle \mid \rangle$ to be the function $q\colon V \longrightarrow F$ defined by $q(v) = \langle v|v\rangle$. Assume that $2 \neq 0$ in F (equivalently, 2 is invertible in F) and prove that the bilinear form can be recovered from q.

2. Let A and A' be real symmetric matrices such that $A' = P^T AP$ for some invertible P. Must A and A' have the same rank?

3. Prove that the only orthogonal, symmetric, positive definite real matrix is the identity.

4. Prove that the eigenvalues of a hermitian matrix are real. (In particular, the eigenvalues of a real symmetric matrix are real.)

5. Prove that a real symmetric matrix is positive definite if and only if its eigenvalues are all positive.