MAT 631 — HOMEWORK 10

DUE ON TUESDAY 13 NOVEMBER

1. Exhibit all Sylow 2- and Sylow 3-subgroups for D_{12}. (Hint for $p = 2$: recall that the center is $Z(D_{12}) = \langle \sigma^3 \rangle$, generated by an element of order 2. Hint for $p = 3$: they are cyclic.) In each case compute $n_p(D_{12})$ so you know you have the right number of subgroups.

2. Exhibit all Sylow 3-subgroups for S_4.

3. Prove that a group of any of the following orders is not simple.

(a) $459 = 3^3 \cdot 17$

(b) $2907 = 3^2 \cdot 17 \cdot 19$.

(c) $6545 = 5 \cdot 7 \cdot 11 \cdot 17$

(d) $300 = 2^2 \cdot 3 \cdot 5^2$

4. Prove that a group of order 105 has *two* (non-trivial, proper) normal subgroups.

5. How many elements of order 7 are there in a simple group of order 168?