MAT 631 — HOMEWORK 2

DUE ON TUESDAY 11 SEPTEMBER

1. Suppose that $a^2 = 1$ for every element a in a group G. Prove that G is Abelian.

2. Consider the presentation

$$D_{2n} = \langle \sigma,\ \tau \mid \sigma^n = 1,\ \tau^2 = 1,\ \sigma\tau = \tau\sigma^{-1} \rangle\,.$$

(a) Show that D_{2n} is generated by two elements of order 2, namely τ and $\tau\sigma$.

(b) Deduce that D_{2n} has another presentation

$$D_{2n} = \langle a,\ b \mid a^2 = b^2 = (ab)^n = 1 \rangle\,.$$

(Derive the relations on $a = \tau$ and $b = \tau\sigma$ from those on σ and τ, and vice versa.)

3. Define permutations

$$\sigma: \quad 1 \mapsto 3, \quad 2 \mapsto 4, \quad 3 \mapsto 5, \quad 4 \mapsto 2, \quad 5 \mapsto 1$$

$$\tau: \quad 1 \mapsto 5, \quad 2 \mapsto 3, \quad 3 \mapsto 2, \quad 4 \mapsto 4, \quad 5 \mapsto 1$$

Find the cycle decompositions of σ, τ, σ^2, τ^2, $\sigma\tau$, and $\tau\sigma$.

4. Find the order of the permutation $(1\ 8\ 10\ 4)(5\ 7)(6\ 9)$.

5. Prove that the order of a permutation σ is the least common multiple of the lengths of the cycles in its cycle decomposition. Find all n such that S_5 contains an element of order n.

6. (bonus) Consider

$$Y = \langle u,\ v \mid u^4 = v^3 = 1,\ uv = v^2u^2 \rangle\,.$$

Prove that Y is the trivial group. (Hint: consider uvu^2 and vuv.)