MAT 532 — HOMEWORK 9

DUE ON THURSDAY 15 NOVEMBER

1. Set

$$A = \begin{pmatrix} -1 & 2 & 1 \\ 2 & -1 & -1 \\ 2 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(a) Use the classical Gram-Schmidt algorithm to transform the columns of A into an orthonormal set in $\mathbb{R}^4$ (with the standard inner product).

(b) Give the QR decomposition corresponding to your answer above.

(c) (OMIT) Repeat, using the modified G-S algorithm. (No QR decomposition necessary.)

(d) Find the least squares solution to $Ax = b$, where $b = (2, 5, -1, -4)^T$.

2. Apply the classical Gram-Schmidt process to the columns of

$$B = \begin{pmatrix} i & 0 & 0 \\ 1+i & 1 & 0 \\ 1 & 1+i & i \end{pmatrix}$$

to get an orthonormal basis for $\mathbb{C}^3$ (with respect to the standard inner product).

3. Consider the matrix

$$C = \begin{pmatrix} 1 & 1 & 1 \\ 10^{-3} & 10^{-3} & 0 \\ 10^{-3} & 0 & 10^{-3} \end{pmatrix}.$$

(a) Apply the classical Gram-Schmidt process to the columns of C, using the standard inner product.

(b) Repeat part (a), this time using 3-digit floating point arithmetic. Is the result an (approximately) orthonormal set?