MAT 532 — HOMEWORK 8

DUE ON THURSDAY 8 NOVEMBER

1. Consider the following orthogonal functions in $\mathscr{C}([-1,1])$ (with respect to the standard inner product $\langle f|g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$).

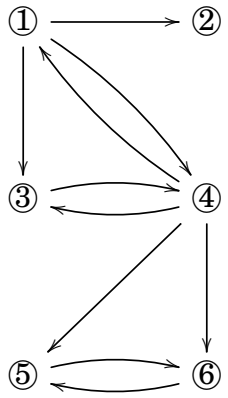
$$\left\{f_1 = x^2 \text{ and } f_2 = x\right\},$$

(a) Find another polynomial $f_3(x) = ax^2 + bx + c$ so that $\{f_1, f_2, f_3\}$ is an orthogonal set. (Hint: solve a system of equations.)

(b) Convert the resulting set to an orthonormal set.

(c) Determine the Fourier expansion of the constant function $g(x) = 1$ with respect to this basis. (Make sure to use the orthonormal basis, not the original functions f_i.)

2. Consider the following schematic for a website:



(a) Write down the "hyperlink matrix" H for this website, with entry

$$h_{ij} = \frac{\#\text{ links } j \longrightarrow i}{\text{total \# links from } j}.$$

(This is the transpose of the matrix from HW #4.)

(b) Write down the matrix S which accounts for dangling nodes.

(c) Write down the Google matrix $G = \alpha S + (1-\alpha)\mathbb{1}$ for $\alpha = 0.85$ and $\alpha = 0.45$.

(d) Compute G^2, G^5 and G^{10} for both matrices in part (c) above. (Use MATLAB or Wolfram | Alpha or something.) Which is converging more quickly to the steady state?