MAT 532 — HOMEWORK 7

DUE ON THURSDAY 25 OCTOBER

Note: We may not get far enough for you to be able to do part (c) of the second and third problem by the due date. I will let you know if they should be postponed to the next problem set.

1. Determine which of the following defines an inner product on $\mathbb{R}^3$. For each one that is, check the defining properties. For each one that fails, give an explicit example (actual numbers) of the failure of one of the defining properties.

(a) $\langle x \mid y\rangle = x_1y_1 + x_3y_3$

(b) $\langle x \mid y\rangle = x_1y_1 - x_2y_2 + x_3y_3$

(c) $\langle x \mid y\rangle = 2x_1y_1 + x_2y_2 + 4x_3y_3$

(d) $\langle x \mid y\rangle = x_1^2y_1^2 + x_2^2y_2^2 + x_3^2y_3^2$

2. Consider orthogonal (with respect to the standard inner product) vectors in $\mathbb{R}^4$

$$\left\{ x_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 3 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}, x_3 = \begin{pmatrix} -3 \\ -3 \\ 3 \\ 0 \end{pmatrix} \right\}.$$

(a) Find a fourth non-zero vector x_4 so that $\{x_1, x_2, x_3, x_4\}$ is an orthogonal set. (Hint: solve a system of equations.)

(b) Convert the resulting set to an orthonormal basis for $\mathbb{R}^4$. How many possible answers are there to this part?

(c) Determine the Fourier expansion of $y = (2,2,2,2)^T$ with respect to this basis. (Make sure to use the orthonormal basis, not the original vectors x_i.)

3. Repeat parts (a), (b) and (c) of the previous problem for

$$\left\{ A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 2 \\ 1 & -2 \end{pmatrix}, A_3 = \begin{pmatrix} 0 & 2 \\ -2 & 1 \end{pmatrix} \right\}$$

with respect to the inner product $\langle A \mid B\rangle = \operatorname{trace}(A^TB)$ on the vector space of 2×2 matrices. (Recall that this induces the Frobenius norm.) For part (c), use

$$B = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}.$$