

MAT 532 — HOMEWORK 6

DUE ON THURSDAY 18 OCTOBER

1. Consider the matrix $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 7 \\ 1 & 2 & 3 & 6 \end{pmatrix}$. You should check (but not hand in) that A has rank 2. Let B be the 3×2 matrix formed from the basic columns of A , and C the 2×4 matrix of nonzero rows of E_A .

- (a) Check that $A = BC$. Formulate a conjecture, related to the fact I asserted in class that a matrix D has rank one if and only if $D = CR$ for a column C and a row R .
- (b) Check that $B^T A C^T$ is nonsingular. Will this always happen?
- (c) Compute

$$A^\dagger = C^T (B^T A C^T)^{-1} B^T.$$

This is the *pseudoinverse* of A .

- (d) (Not to hand in, but strongly suggested to do.) Choose a random 3×1 matrix b , set $z = A^\dagger b$, and verify that z is a solution to the normal equations $A^T A x = A^T b$.

See (4.5.20).

2. Let $x = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 1 \end{pmatrix}$. Find the 1-norm, the 2-norm, and the ∞ -norm of x . Repeat with $y = \begin{pmatrix} 3-i \\ 1+i \\ 2 \\ 2i \end{pmatrix}$.

3. Find the 1-norm, the 2-norm, the ∞ -norm, and the Frobenius norm of the following matrices.

(a) $A = \begin{pmatrix} 3 & -1 \\ -3 & 1 \end{pmatrix}$

(b) $B = I_3$

(c) $C = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(d) $D = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ -1 & 1 & 2 \end{pmatrix}$