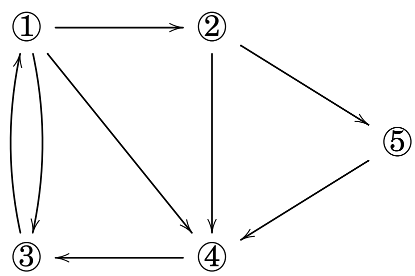
MAT 532 — HOMEWORK 4

DUE ON THURSDAY 27 SEPTEMBER

1. Consider a website with five pages, which link to each other according to the following diagram:



(a) Write down the "link matrix" C for this tiny Internet, with entry $c_{ij} = \frac{\text{\# links } i \longrightarrow j}{\text{total \# links from } i}$.

(b) Compute a high power of C somehow (Wolfram | Alpha is good for this – let me know if you need help). I suggest at least C^{12}, and higher powers are better. Interpret the result.

2. Suppose you are given that

$$A = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \qquad \text{and} \qquad A^{-1} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}.$$

(a) Compute $\hat{A}^{-1}$, where $\hat{A}$ is the matrix obtained by setting the $(3,2)$ entry of A to 0.

(b) Compute $\hat{A}^{-1}$, where $\hat{A}$ is obtained by adding 1 to the bottom right entry of A.

3. Compute the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ & 1 & 5 & 6 \\ & & 1 & 7 \\ & & & 1 \end{bmatrix}$$

by writing A as $I - T$, with T upper-triangular, and applying the Neumann series.

4. Suppose A is an $m \times n$ matrix. Write down a formula for $\text{tr}(A^T A)$. (Hint: try it with small examples, say 1×2, 2×1 and 2×2 at first.) If A has real (non-complex) entries, explain why $\text{tr}(A^T A) = 0$ implies $A = 0$.