MAT 532 — HOMEWORK 2

DUE ON THURSDAY 13 SEPTEMBER

1. Use the Gauss-Jordan method to solve the following system.

$$\begin{aligned} 2x - 5y + 4z &= 8 \\ 2x + 0y + 2z &= 4 \\ -1x - 2y + 1z &= 2 \end{aligned}$$

2. Divide the interval $[0,1]$ into five equal subintervals, and apply the finite difference method to approximate a solution to the two-point boundary value problem

$$y''(t) = 8t, \qquad y(0) = 0, \qquad y(1) = 1.$$

3. Consider the following system:

$$\begin{aligned} 10^{-3}x + y &= 1 \\ x - 2y &= 0 \end{aligned}$$

(a) Use 3-digit floating point arithmetic without partial pivoting to solve this system.

(b) Find a system that is exactly solved by your approximate solution in part (a).

(c) Now use 3-digit arithmetic with partial pivoting to solve the same system.

(d) Find a system that is exactly solved by your approximate solution in part (c).

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5. Rank the following three systems in order of worsening condition.

(a)

$$\begin{aligned} 1.001x - y &= .352 \\ x + .0001y &= .657 \end{aligned}$$

(b)

$$\begin{aligned} 1.001x - y &= .352 \\ x + .9999y &= .657 \end{aligned}$$

(c)

$$1.001x + y = .352$$

$$x + .9999y = .657$$

6. Determine the exact solution of the following system:

$$\begin{aligned} 11x + 10y + 14z &= 1.001 \\ 12x + 11y - 13z &= .999 \\ 14x + 13y - 66z &= 1.001 \end{aligned}$$

(You may pretend that your calculator gives you the "exact solution".) Now round the output column to consist of all 1's and again solve the system with exact arithmetic. (You can do these two steps together using the method of HW 1, #2.) Is the system ill-conditioned?