This exam has 6 problems on 7 pages. Name: _____

Show all work for full credit. Score: _____/100

Put a box around your answer.

1. Consider

$$A = \begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix}.$$

(a) (10 pts) Compute an LU decomposition for A.

(b) (15 pts) Compute a QR decomposition for A, using the modified Gram-Schmidt orthonormalization procedure.

2. (20 pts) Compute a singular value decomposition for

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}.$$

3. (10 pts) Of the two matrices

$$A = \begin{pmatrix} 1.001 & 1 \\ 1 & .999 \end{pmatrix} \qquad \text{and} \qquad A = \begin{pmatrix} 1.001 & 1 \\ -1 & .999 \end{pmatrix},$$

one is ill-conditioned, and one is well-conditioned. Determine which is which by computing the condition numbers with respect a convenient norm.

4. (15 pts) Compute the reduced row-echelon form for

$$A = \begin{pmatrix} 2 & -4 & -8 & 6 & 3 \\ 0 & 1 & 3 & 2 & 3 \\ 3 & -2 & 0 & 0 & 8 \end{pmatrix},$$

using 3-digit floating point arithmetic.

5. (15 pts) Find the parabola $y = ax^2 + bx + c$ that best fits the following data (which are not on a parabola!). Compute the error $\epsilon = \|Ax - b\|_2$ for your least-squares solution.

x	0	1	2	3	4
y	-10	4	8	5	-2

6. (15 pts) Consider the inner product space V of continuous functions on $[-\pi, \pi]$, with inner product defined by

$$\langle f \mid g \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)g(x)\, dx.$$

The functions $f_1(x) = 1$ and $f_2(x) = x$ are orthogonal in V. Find a third quadratic polynomial $f_3(x)$ so that $\{f_1, f_2, f_3\}$ is an orthogonal set in V, and compute the Fourier expansion of $g(x) = \cos x$ with respect to this set. (Hint: the antiderivatives of $\cos x$, $x\cos x$, and $x^2 \cos x$ are, respectively, $\sin x$, $x\sin x + \cos x$, and $x^2 \sin x + 2x\cos x - 2\sin x$.)