Climbing the Chimney

Stories about the times I got a belay from UNL Math

Graham Leuschke

Syracuse University

Lincoln — 29 April 2023

Theme

The UNL Mathematics Department has been inextricably intertwined with my life – professionally, mathematically, personally – for nearly three decades.

Background

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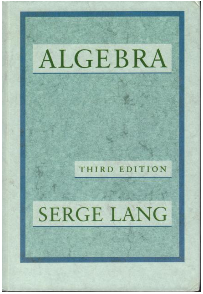

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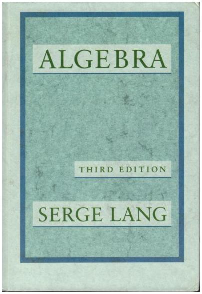


§10. GALOIS COHOMOLOGY

Let G be a group and A an abelian group which we write additively for the general remarks which we make, preceding our theorems. Let us assume that G operates on A, by means of a homomorphism $G \to \mathrm{Aut}(A)$. By a **1-cocycle** of G in A one means a family of elements $\{\alpha_\sigma\}_{\sigma \in G}$ with $\alpha_\sigma \in A$, satisfying the relations

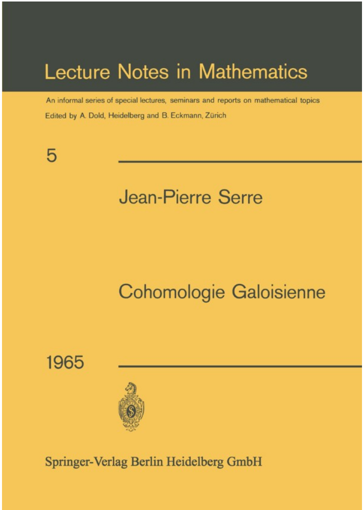
$$\alpha_\sigma + \sigma\alpha_\tau = \alpha_{\sigma\tau}$$

for all $\sigma, \tau \in G$. If $\{\alpha_\sigma\}_{\sigma \in G}$ and $\{\beta_\sigma\}_{\sigma \in G}$ are 1-cocycles, then we can add them to get a 1-cocycle $\{\alpha_\sigma + \beta_\sigma\}_{\sigma \in G}$. It is then clear that 1-cocycles form a group, denoted by $Z^1(G, A)$. By a **1-coboundary** of G in A one means a family of elements $\{\alpha_\sigma\}_{\sigma \in G}$ such that there exists an element $\beta \in A$ for which $\alpha_\sigma = \sigma\beta - \beta$ for all $\sigma \in G$. It is then clear that a 1-coboundary is a 1-cocycle, and that the 1-coboundaries form a group, denoted by $B^1(G, A)$. The factor group

$$Z^1(G, A)/B^1(G, A)$$

is called the **first cohomology group** of G in A and is denoted by $H^1(G, A)$.

Background


Lecture Notes in Mathematics
An informal series of special lectures, seminars and reports on mathematical topics
Edited by A. Dold, Heidelberg and B. Eckmann, Zürich
5
Jean-Pierre Serre
Cohomologie Galoisienne
1965
Springer-Verlag Berlin Heidelberg GmbH

Background

Background

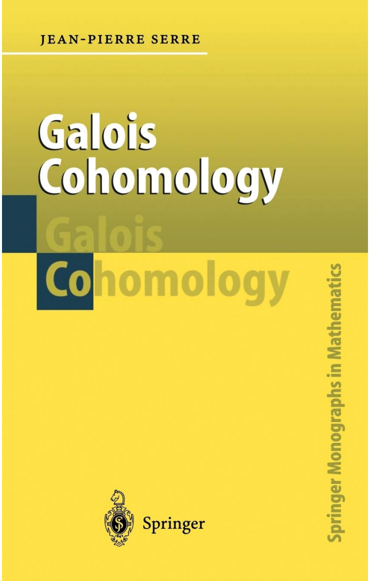
Date: Tue, 1 Feb 1994 09:36:33 +0100
From: Jean-Pierre Serre <Jean-Pierre.Serre@ens.fr>
To: gleuschk@reed.edu
Subject: Re: hello...

Dear M.Graham,
I don't know of any project of translating in English "Cohomologie Galoisienne".
As for new results in the field, of course there have been many ! Some may be found in the recent book of Platonov and Rapinchuk (in English) called Alg.Groups and Number Theory. I shall myself give a report on other such things in the next Bourbaki seminar.
About CG : I am preparing a new revised edition, which will give a few complements. But it is far from ready yet (and it will be in French).
Best
J-P.Serre

Date: Tue, 1 Feb 1994 17:13:52 +0100
From: Jean-Pierre Serre <Jean-Pierre.Serre@ens.fr>
To: gleuschk@reed.edu
Subject: Re: hello...

Dear M.Gleusch,
There is a point in your letter that I find ambiguous. You say that your are starting my ... project "on exactly that topic" > What topic? Do you mean:
Galois cohomology
or
translating into English Cohomologie Galoisienne ?
I hope the first interpretation is correct; there is little point in translating mathematics from French to English.
Yours
J-P.Serre

Background


JEAN-PIERRE SERRE
Galois
Cohomology
Galois
Cohomology
Springer
Springer Monographs in Mathematics

First connection

January 10, 1994

Commutative Algebra with a view toward Algebraic Geometry

by

David Eisenbud
Dept. of Mathematics
Brandeis University, Waltham MA 02254
eisenbud@math.brandeis.edu

To Appear as a Springer-Verlag Graduate Text, approximately September 1994

First connection

Date: Fri, 16 Sep 1994 12:25:14 -0400 (EDT)
From: David Eisenbud <de@oscar.math.brandeis.edu>
To: "Graham J. Leuschke" <gleuschk@reed.edu>
Subject: Re: thanks and questions....

Dear Mr. Leuschke,

It's hard to do uniform justice to a request like yours -- in a way, a conversation is an easier format. Still, some places that have extremely strong algebra programs but are not in the top ten do come to mind. For obvious reasons I'm partial to Brandeis. U. of Michigan is first-rate (Hochter, Dolgachev, others...). Purdue probably has the largest (and best) commutative algebra group in the whole world at the moment. Not quite at this level but also interesting is U of Nebraska at Lincoln (especially Roger Wiegand, but quite a few others are there too). Is this the sort of thing you had in mind?

Regards,
David Eisenbud

First connection

Ann Sanfedele via Wikipedia

Visiting

Visiting

Visiting

TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 337, Number 1, May 1993

CHARACTERIZATION OF COMPLETIONS OF UNIQUE FACTORIZATION DOMAINS

RAYMOND C. HEITMANN

ABSTRACT. It is shown that a complete local ring is the completion of a unique factorization domain if and only if it is a field, a discrete valuation ring, or it has depth at least two and no element of its prime ring is a zerodivisor. It is also shown that the Normal Chain Conjecture is false and that there exist local noncatenary UFDs.

Visiting

Lemma 4. *Let* (T, M) *be a complete local ring,* R *an* N*-subring of* T, $c \in R$, I *a finitely generated ideal of* R, *and* $c \in IT$. *Then there exists S,* $R \subset S \subset T$, *such that* S *is an* N*-subring of* T, $|S| = |R|$, *prime elements in* R *are prime in* S, *and* $c \in IS$.

Arriving

Arriving

Definition

Let G be a finite group. Let J be the left ideal of the integral group ring $\mathbb{Z}[G]$ generated by all sums of the form $\sum_{h \in H} h$, as H runs over all proper subgroups of G.

The Scharlau invariant of G is the non-negative integer generating the principal ideal $J \cap \mathbb{Z}$.

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Fact

This number is either 0, 1, p, or p^2 for some prime p.

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Fact

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Question

For $G = \mathrm{SL}(2, 17)$ it is either 17 or 17^2. Which?

Entering

Entering

Definition

Let R be a local ring. We say that R has finite Cohen-Macaulay type if it admits only finitely many indecomposable maximal Cohen-Macaulay modules (up to isomorphism).

Entering

CONJECTURE

a) R has finite CM-representation type iff the m-adic completion $\hat{R}$ has finite CM-representation type.

b) R has finite CM-representation type iff $R \otimes_k \bar{k}$ has finite CM-representation type, where $\bar{k}$ denotes the algebraic closure of k.

One might ask similar questions in the mixed characteristic case.

Entering

2.9. THEOREM. *Let* $(R, \mathbf{m})$ *be a local ring with Henselization* R^h *and* $\mathbf{m}$*-adic completion* $\hat{R}$. *Consider the following three statements*:

(a) *R has finite CM type.*

(b) *R^h has finite CM type.*

(c) *$\hat{R}$ has finite CM type.*

Then:

(1) *Statement* (c) $\Rightarrow$ (b) $\Rightarrow$ (a).

(2) *If* $\dim(R) \le 1$ *then* (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c).

(3) *If R is Cohen–Macaulay and R_P is Gorenstein for each prime ideal* $P \neq \mathbf{m}$, *then* (a) $\Leftrightarrow$ (b).

(4) *If R is Cohen–Macaulay and $\hat{R}$ has an isolated singularity, then* (a) $\Leftrightarrow$ (b) $\Leftrightarrow$ (c).

(5) *If R is Cohen–Macaulay and R^h is excellent and has a canonical module, then* (b) $\Leftrightarrow$ (c).

Entering

PROPOSITION 3 (AUSLANDER [**3**, **5**]). *A complete, Cohen-Macaulay R-algebra is an isolated singularity if it is of finite Cohen-Macaulay type.*

Ascent of Finite Cohen–Macaulay Type

Graham Leuschke and Roger Wiegand[1]

Department of Mathematics, University of Nebraska, Lincoln, Nebraska 68588

Communicated by Craig Huneke

Received October 23, 1999

In 1987 F.-O. Schreyer conjectured that a local ring R has finite Cohen–Macaulay type if and only if the completion $\widehat{R}$ has finite Cohen–Macaulay type. We prove the conjecture for excellent Cohen–Macaulay local rings and also show by example that it can fail in general.

Key Words: finite Cohen–Macaulay type; canonical module.

Interlude

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Away, and Return

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Setup

Let k be a field and let $X = [x_{ij}]$ be a square matrix of indeterminates x_{ij}.

Recall: $\mathrm{adj}(X)$ is the "classical adjoint" of X

$$X \ \mathrm{adj}(X) = \det(X) \cdot I_n = \mathrm{adj}(X) \ X$$

Away, and Return

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Question (G. Bergman)

Can this factorization be refined? That is, can $\mathrm{adj}(X)$ be written as $\mathrm{adj}(X) = YZ$ for noninvertible square matrices Y and Z?

Away, and Return

Translation

Bergman's Question is equivalent to a question about maximal Cohen-Macaulay modules over the hypersurface ring $k[x_{ij}]/(\det(X))$ (namely, the cokernel of the matrix $\operatorname{adj}(X)$).

Specifically: Does the cokernel of $\operatorname{adj}(X)$ appear as the middle term of a short exact sequence of MCM modules?

Away, and Return

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Specifically: Does the cokernel of $\mathrm{adj}(X)$ appear as the middle term of a short exact sequence of MCM modules?

Answer

Yes, when the size of the matrix is even. In fact there is one factorization for every skew-symmetric matrix over $k[x_{ij}]$.

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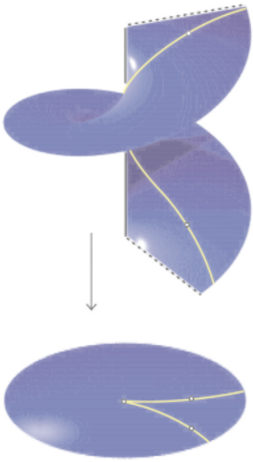

Herwig Hauser

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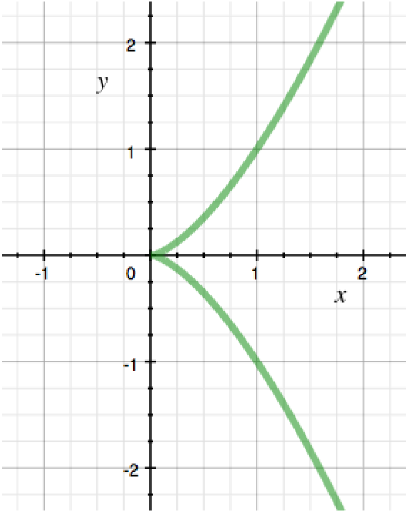

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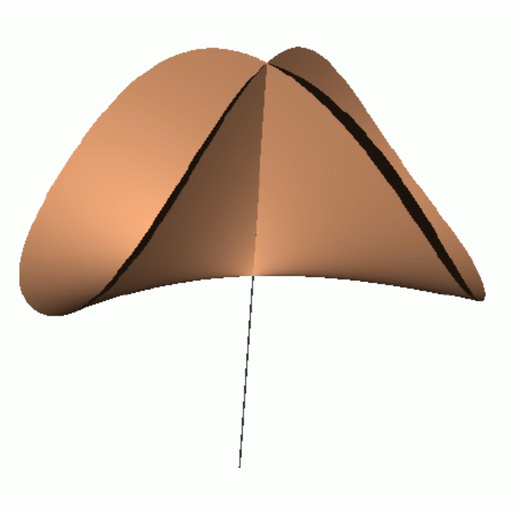

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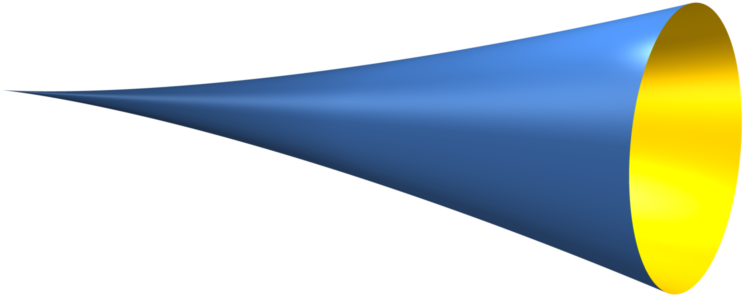

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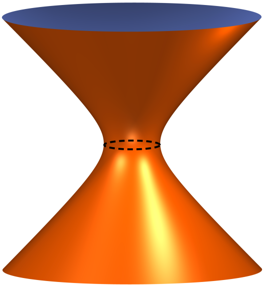

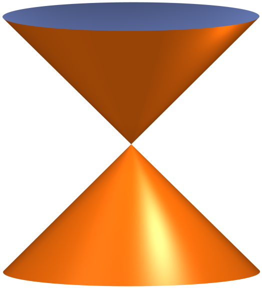

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